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Thermal networks: why the chip has more than one time constant.

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Abstract

This analytical paper derives a two-node thermal network from energy conservation and compares its modal time constants with the branch RC products. An exactly specified example has two identical branch products of one second, yet its system time constants are 0.382 and 2.618 seconds. A reproducible step-response calculation demonstrates the resulting error of a single-time-constant approximation. The result establishes a model-selection principle: thermal dynamics must be evaluated for the coupled network, the measured output and the intended time horizon. It does not establish parameters for a particular processor or demonstrate cooling-energy savings.

1 Scope and thermal coordinates

A processor package contains several paths for heat transport and several volumes that store heat. A compact thermal network replaces this distributed structure with a finite set of temperatures, thermal capacitances and conductances. It is useful when those states resolve the dynamics required by the application. Adding states alone does not establish accuracy: parameters, boundary conditions and observed quantities must also be defined.

The following example uses two storage nodes connected to a fixed-temperature bath. The first represents a heat-source region and the second a downstream package region. Temperature rises θ_1 and θ_2 are measured relative to the constant bath temperature. Both start at zero. All parameters are illustrative and constant; lateral spreading, radiation, fluid transport delay and temperature-dependent material properties are excluded.

2 Derivation from the energy balance

Let C_1 and C_2 denote thermal capacitances in J/K. R_1 is the resistance between the nodes and R_2 the resistance from node two to the bath, both in K/W. Electrical dissipation P enters node one as heat. The inter-node heat flow is $(\theta_1 - \theta_2)/R_1$. It leaves the first storage volume and enters the second with exactly the opposite sign.

Every term in the nodal equations has units of watts. Adding both balances cancels the internal heat flow and gives $d(C_1\theta_1 + C_2\theta_2)/dt = P - \theta_2/R_2$. This cancellation is an immediate consistency check. At steady state the bath removes the entire input: $\theta_2 = PR_2$ and $\theta_1 = P(R_1 + R_2)$. The stored sensible-heat change is $C_1\theta_1 + C_2\theta_2$, measured in joules.

$$C_1\dot{\theta}_1 = P - \frac{\theta_1 - \theta_2}{R_1}, \qquad C_2\dot{\theta}_2 = \frac{\theta_1 - \theta_2}{R_1} - \frac{\theta_2}{R_2}. \tag{1}$$

Coupled nodal balances for a fixed-temperature bath.

Research origin: Memo 059 (2026-04-10). Online edition: `https://arflo.io/research/thermal-networks-and-timescales/`.

3 Local RC products and system modes

For one isolated first-order storage element connected through one resistance, $\tau = RC$ is the response time constant. In a coupled ladder, changing either temperature affects both balances. The system time constants therefore follow from the eigenvalues of the complete state matrix A, rather than from separately multiplying each layer resistance by its nearby capacitance.

Choose $C_1 = C_2 = 10$ J/K and $R_1 = R_2 = 0.1$ K/W. Both branch products equal one second. The resulting matrix is A = [[-1, 1], [1, -2]] s^{-1}. Its characteristic polynomial is $\lambda^2 + 3\lambda + 1$, with coefficients carrying the corresponding inverse-time units. The eigenvalues are $(-3 \pm \sqrt{5})/2$ s^{-1}. The two positive decay times are consequently 2.618034 s and 0.381966 s. Neither equals one second.

$$\dot{\boldsymbol{\theta}} = A\boldsymbol{\theta} + \begin{bmatrix} P/C_1 \\ 0 \end{bmatrix}, \qquad \tau_k = -\lambda_k^{-1}. \tag{2}$$

For this grounded passive network the eigenvalues are negative and the modal time constants are positive.

4 A reproducible power-step example

Apply a constant 100 W step at time zero, retaining the parameters above. The steady-state temperature rises are 20 K at the source and 10 K downstream. With zero initial rise, the exact linear-system solution is $\theta(t) = \theta\infty$ - exp(At)$\theta\infty$. This expression can be evaluated by diagonalizing the two-by-two matrix or by a matrix-exponential routine.

For comparison, consider a one-state approximation with the correct final source rise but an arbitrarily selected one-second time constant: $\hat{\theta}_1(t) = 20[1 - \exp(-t/1\ s)]$ K. At one second it predicts 12.642 K instead of 6.993 K. The 5.649 K discrepancy is an error of this explicitly chosen approximation, not a universal error bound for one-state models. A different fitted time constant could improve one interval while retaining error elsewhere.

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}_\infty - e^{At}\boldsymbol{\theta}_\infty, \qquad \boldsymbol{\theta}_\infty = \begin{bmatrix} 20 \\ 10 \end{bmatrix} \text{ K}. \tag{3}$$

Exact zero-initial-condition step response for the specified constant parameters.

Table 1: Calculated temperature rises for the illustrative 100 W step.

Time / s	**Source / K**	**Downstream / K**	**One-state source / K**
0.1	0.953	0.045	1.903
0.5	4.064	0.789	7.869
1.0	6.993	2.134	12.642
2.0	11.170	4.555	17.293
5.0	17.194	8.266	19.865
10.0	19.584	9.743	19.999

5 What measurements can establish

A Foster representation expresses a measured thermal response as a sum of exponential terms. A Cauer representation uses a ladder topology. Equivalent realizations can describe the same external thermal impedance; a Foster model is not inherently a one-state model. Internal nodes gain physical interpretation only when the network and its parameters have been related to the device and its boundary conditions. These distinctions are documented in semiconductor thermal-model references [1, 2].

For identification, record the actual dissipated-power profile, reference temperature, sensor locations, timestamps and measurement bandwidth. Fit on one set of transients and evaluate different transients separately. Report the temperature error over the time horizon of interest, including the earliest resolved response and the approach to equilibrium. A sensor too slow to resolve the fast mode cannot establish that the mode is absent.

The numerical example is reproducible without any device-specific data. Form $C = \mathrm{diag}(10, 10)$ and $K = [[10, -10], [-10, 20]]$, calculate $A = -C^{-1}K$, and solve $K\theta\infty = [100, 0]^{\mathsf{T}}$. Evaluate the displayed matrix-exponential expression at each tabulated time. Checks are $\theta(0) = 0$, the initial source slope $P/C_1 = 10$ K/s, the final rise [20, 10] K and zero residual in both steady-state balances. These checks test arithmetic and implementation consistency; they do not replace comparison against an independently measured physical system.

6 Implications and limits

The calculation establishes why thermal model order should follow the input-output dynamics required by the task. A model adequate for minute-scale outlet-temperature planning can still omit a mode important for a subsecond source-temperature estimate. Conversely, retaining every material layer is unnecessary when its contribution to the relevant output is demonstrably negligible over the assessed operating range.

The illustrative network does not resolve three-dimensional hotspots or determine actual processor thermal margins. Variable coolant temperature, flow-dependent conductance, sensor filtering and multiple heat sources require extensions and new validation. The defensible conclusion is narrower and useful: evaluate coupled modes and the selected output before assigning a single thermal time constant.

Conclusion

A local RC product describes an element. The transient seen by a sensor is a property of the coupled network.

References

[1] Nexperia (2021). AN11261: RC Thermal Models, rev. 5.0, sections 3-5.. `https://assets.nexperia.com/documents/application-note/AN11261.pdf`.

[2] Gerstenmaier, Kiffe & Wachutka (2007). Combination of Thermal Subsystems Modelled by Rapid Circuit Transformation. THERMINIC; arXiv:0801.0817.. `https://arxiv.org/abs/0801.0817`.